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MIXING LENGTHS AND ENTRAINMENT RATIOS IN LOW-VELOCITY JET-PUMPS

by

J. E. Nowrey and Arnold Kivnick

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The work on ducted jets carried out earlier under this contract dealt with high-velocity jets discharging into large ducts. The jet velocities were about 600 ft. per second, and the duct area was 16 times as large as the jet area. The problem of cooling a gas-turbine by using the momentum of the hot exhaust gases to induce the flow of cooling air indicated that low-velocity jets (say about 100 ft. per second) should be investigated also. The entrainment ratio desired for turbines is much lower than the values found with the high-velocity jets.

The present investigation was undertaken in order to measure the mixing length for a new low-velocity jet-duct system and to compare this value with that found for the high-velocity jets. It was also desirable to see how the new mixing length depended on the jet velocity.

The earlier work with the 600 ft. per second jets had shown that the mixing length was about seven duct diameters. That is, the primary and secondary air had mixed sufficiently in that distance to provide a flat velocity profile across the duct. At that location, the static pressure in the duct was at a maximum. These observations were true for non-isothermal systems (3, 4) as well as for isothermal ones (2). In each case the entrainment ratio (weight of entrained air to weight of primary air) was about 3.5.

Some earlier tests had been made with lower entrainment ratios (2). The flow of secondary air was reduced somewhat by the use of screens

at the secondary air entrance, or by restricting the total flow at the duct discharge. In both cases the mixing length was found to decrease from seven diameters to about five diameters. It was observed that the mixing length was about proportional to the entrainment ratio.

For the new low-velocity, low-entrainment tests, the air-supply system described by Alexander (1) was used. The flow metering system was the same as that used by Danielson (3). The jet and duct designs were changed to provide the necessary low entrainment. Ideally, the jet could have issued from a standard, faired nozzle. However, a short length of 2-inch standard steel pipe was selected instead. This simulated the design of a home-made jet pump and also provided a considerable saving in time. The edge of the "nozzle" was tapered to a sharp-edge (with the inside diameter remaining constant). The duct had an inside diameter of 2.75 inches and was provided with a faired entry of the type used for subsonic flow. The ratio of the initial jet area to the minimum annular, secondary-air area was 1 to 0.33 (compared to about 1 to 15 for the high-velocity jets). Holes were located at 1-inch intervals along the duct to permit static pressure measurements and impact tube traverses.

During any run, all the data were obtained in one session. The principal data consisted of measurements of the primary air-flow, the duct static pressure profile (along the axis), and the duct impact-tube traverses at various cross-sections along the duct. Each impact-tube traverse consisted of about 15 readings along one radius of the duct.

The entrained-air flow was calculated by difference. The total air flowing in the duct was found by a graphical evaluation of the impact-tube traverses, weighting each value according to the annular area.

represented by that reading. The primary air fraction of this total flow was subtracted to give the secondary air flow. The calculated total air flow varied from the mean by about ± 10 percent from station to station along the duct length. Since the test conditions did not change, the variability might be attributed to differences in the degree of turbulence along the duct. The same phenomenon was noted earlier by Henze (4) and by Danielson (3).

Although the total air flow varied from the mean by ± 10 percent, the secondary air flow varied even more. The secondary air flow was about one third of the total flow, hence the secondary air flow varied from the mean by as much as ± 30 percent. This fact must be kept in mind when Table I is examined.

Table I is a summation of the entrainment calculations for the tests. Runs were made at three primary-jet velocities: 50, 100, and 250 feet per second. The entrainment ratio is seen to be about 0.3.

Table I

Air Entrainment by a Slow Jet

Initial Velocity of Jet, ft./sec.	Entrainment Ratio	
	Average	At Maximum Static Pressure
47.4	0.36	0.29
97.5	0.3[illegible]	0.27
249	0.32	0.28

There appears to be a slight dependence of entrainment on jet velocity, but this is certainly not significant in view of the uncertainty in the secondary air flow. It is concluded that the entrainment ratio is independent of the jet velocity.

Considering the mixing length, an unexpected fact was discovered. The mixing length is the distance necessary for a flowing, non-mixed stream to become completely mixed. The point of final complete mixing may be determined by static pressure measurements, or by velocity profile determinations. For the early high-velocity jets, both methods were used, and the values agreed. The profile typical of flow in a pipe occurred first at a distance of seven duct diameters from the nozzle (2,3,4). The static pressure was a maximum at the same location. However, for the low velocity jets, the two criteria of complete mixing gave distinctly different values for the mixing length. The mixing length was five duct diameters as determined by static pressures; it was ten diameters according to the velocity profiles. This was true at all three test velocities. This of course means that a stable flow near the duct wall occurs before mixing is complete in the main bulk of fluid.

Other workers have reported mixing lengths of from four to ten duct diameters, the value depending on the experimental design. The apparatus design of course is of prime importance. If a faired jet-nozzle had been used for the present tests, it is entirely possible that different entrainments and different mixing lengths would have resulted. Nevertheless, it seems that a duct having a length-to-diameter ratio of ten is sufficient to obtain complete mixing of a jet with entrained air, regardless of whether the jet velocity is high or low, whether the entrainment ratio is high or low, and probably whether the nozzle is faired or a simple pipe. In some cases a shorter duct will be sufficient.

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